

Multiple hadron production in e^+e^- annihilation induced by heavy primary quarks. New analysis

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Abstract

In this paper we present an analysis of the multiple hadron production induced by primary heavy quarks in e^+e^- annihilation with the account of most complete and corrected experimental data. In the framework of perturbative QCD, new theoretical bounds on the asymptotically constant differences of the multiplicities in processes with light and heavy quarks are given.

1 Introduction

As it is already known in classical theory, the more is the mass of a charged particle, the less intensive is the radiation from it. In quantum field theory, in particular, QCD, this circumstance leads to a number of impressive effects caused by heavy quarks, as, for instance, the effect of leading mesons which contain c - and b -quarks in e^+e^- annihilation [1]. If we assume that the “radiation” in QCD is the radiation of gluons, which “materialize” afterwards in form of detected hadrons, it is natural to expect, that in events induced by heavy quarks, the hadron multiplicity (except for decay products of the leading quarks) is smaller than in analogous events triggered by light

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quarks. So natural may be an expectation that the difference between these multiplicities has to disappear at high enough energies.

However, it is not always the case. For instance, even in QED, the difference between cross sections (times the c.m. energy squared) of fermion-antifermion pairs of different masses produced by two photons, does not disappear with increase of energy, but tends to a constant which depends on the masses of the final fermions [2]. The measurements of hadron multiplicities in e^+e^- annihilation associated with the primary quarks of definite flavors (in practice, u , d and s quarks are assumed to be massless) were carried out at lepton colliders with the collision energy of 29 GeV and more, in particular, at the SLC, KEK, LEPI and LEP II.

In view of a great interest caused by these experiments (see below), and due to the presence of competing theoretical predictions, there is no doubts that similar measurements will be also done at future linear colliders, such as CLIC or ILC [3]. The first attempt to look for such effects was done in the framework of so-called “naïve model”. The essence of this model lies in the fact that an universal mechanism of multiple production of hadrons from some gluon system is adopted, which is insensitive to the quark flavors, while all the difference between processes induced by distinct quark-antiquark pairs arises due to a difference in energy available for the hadron production.

Later on, another approach was accepted appealing directly to the calculational kit of perturbative QCD. However, the estimates obtained this time overestimated the data significantly. For all that, there was a clear indication of the asymptotic constancy of the multiplicity differences from the heavy and light quarks. Not long after, the same quantity, the multiplicity difference in the events with the light and heavy quarks, was calculated more accurately. The results appeared to be strikingly close to the experimental data.

As was already said, the “naïve model” [4, 5] was the first attempt to take into account hadrons produced in addition to decay products of the heavy quark-antiquark pair in e^+e^- annihilation. Later on, it was argued [6] that the difference between multiplicities in events induced by heavy ($Q = c, b$) and light ($l = u, d, s$) quarks,

$$\delta_{Ql} = N_{Q\bar{Q}}(Q^2) - N_{l\bar{l}}(Q^2), \quad (1)$$

tends to a constant value with increase of energy $Q = \sqrt{q^2}$:

$$\delta_{Ql} \rightarrow \delta_{Ql}^{\text{MLLA}} = 2n_Q - N_{l\bar{l}}(m_Q^2). \quad (2)$$

Here (and below) it is assumed that we deal with *mean* multiplicities of *charged* hadrons, and “e” is the base of the natural logarithm ($\ln e = 1$).

The comparison with the data has shown that the “naïve model” describes the data on δ_{bl} up to $Q = 58$ GeV [4, 7, 8, 9], but underestimates the LEP and SLAC data [10, 11, 12]. As for the formula which was obtained at the basis of the co-called “modified logarithmic approximation (MLLA)” (2), it has significantly overestimated both low and high-energy data on δ_{bl} .

Detailed QCD calculations of the difference between associated multiplicities of charged hadron in e^+e^- annihilation were made in [13]. The QCD expressions for δ_{Ql} from Ref. [13] appeared to be in a good agreement with experimental measurements of associated hadron multiplicities in e^+e^- annihilation (see, for instance, [14, 15]).

As we will see below, it is the hadron multiplicity in the light quark events that enables one to calculate the multiplicity differences δ_{Ql} . Recently, the data on average charged multiplicities in $l\bar{l}$ events at different energies corrected for detector effects as well as for initial state radiation were presented in [16]. The corrected multiplicity differences averaged over all presently published results can be found in Ref. [16]:

$$\delta_{bl}^{\text{exp}} = 3.12 \pm 0.14 , \quad (3)$$

$$\delta_{cl}^{\text{exp}} = 1.0 \pm 0.4 . \quad (4)$$

Because of the appearance of these corrected experimental data, a natural necessity has arisen to reconsider our predictions for δ_{bl} with the account of the data on the hadron multiplicity in the light quark events $N_{l\bar{l}}(Q^2)$ as well.

In Section 2 we define all the relevant quantities, since rigorous definitions are not usually presented by other authors. The analytical formula for the hadron multiplicity in e^+e^- annihilation is obtained. The QCD expression for the multiplicity difference is derived in the next section. In Section 4 the upper and lower bounds on δ_{bl} are calculated. In Appendix A the detailed derivation of the evolution equation for the multiplicity in the gluon jet is given. In Appendix B we discuss the connection between our approach and the scheme which uses the concept of the Altarelli-Parisi decay functions. The problems related to the gauge invariance guarantee which appear in perturbative calculations of the “light multiplicity” are considered in Appendix C.

2 Hadron multiplicities in e^+e^- annihilation

The average multiplicity of hadrons in a $q\bar{q}$ event in the process of e^+e^- annihilation is of the form

$$N_{q\bar{q}}^h(Q^2) = 2n_q + \int \frac{d^4k}{(2\pi)^4} \Pi_{\mu\nu}^{ab}(q, k) d_{aa'}^{\mu\alpha}(k) d_{bb'}^{\nu\beta}(k) n_{\alpha\beta}^{a'b'}(k), \quad (5)$$

where $d_{ab}^{\mu\nu}(k)$ is the propagator of the gluon with momentum k . Here and below (a, b) and (a', b') denote color indices.

The subscript q denotes the type of primary quarks. In what follows, the notation $q = Q$ (*heavy quark*) will mean charm or beauty quark, while the notation $q = l$ (*light quark*) designates a pair of u , d or s -quarks which are assumed to be massless. In particular, $N_{ll}(Q^2)$ means the multiplicity of hadrons in light quark events, while $N_{Q\bar{Q}}(Q^2)$ is the multiplicity of hadrons in events when the process is induced by the heavy quark and antiquark of the type Q .

The first term in the r.h.s. of Eq. (5), $2n_q$, is the multiplicity from the fragmentation of the leading quark (antiquark). It is taken from the analysis of data ($2n_c = 5.2$, $2n_b = 11.0$ [6], and $2n_l = 2.4$ [14]). The quantity $n_{\alpha\beta}^{a'b'}(k)$ in the integrand of Eq. (5) is given by the diagram in Fig. 1 in which both the integration in the momentum of the final hadron and averaging in its polarization are assumed. This diagram corresponds to the following analytical expression:

$$\begin{aligned} n_{\alpha\beta}^{a'b'}(k) &= \int \frac{d^3h}{(2\pi)^3 2h_0} \iiint d^4x d^4y d^4z e^{ikx - ih(y-z)} \\ &\times \langle 0 | \left(T I_\alpha^{a'}(x) J_h^+(y) \right) \left(T I_\beta^{b'}(0) J_h(z) \right) | 0 \rangle \\ &= -i \iiint d^4x d^4y d^4z e^{ikx} D_h^-(y-z) \\ &\times \langle 0 | \left(T I_\alpha^{a'}(x) J_h^+(y) \right) \left(T I_\beta^{b'}(0) J_h(z) \right) | 0 \rangle. \end{aligned} \quad (6)$$

Here $J_h(x)$ is the source operator of hadron h (spin indices are omitted), and $I_\alpha^a(x)$ is the gluon (color) current [17],

$$I_\mu^a(x) = i \frac{\delta S}{\delta A_\mu^a(x)} S^*, \quad (7)$$

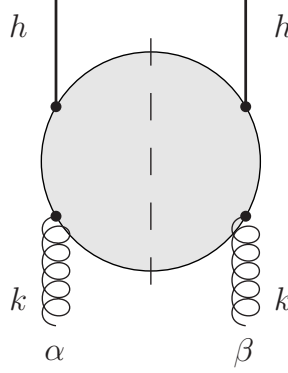


Figure 1: The diagram which describes the average multiplicity of hadrons with the 4-momentum h (solid lines) in the gluon jet with the virtuality k^2 (winding lines).

where $A_a^\mu(x)$ denotes the gluon field. $D_h^-(x)$ is the Pauli-Jordan function [17]:

$$D_h^-(x) = i \langle 0 | h(x) h(0) | 0 \rangle , \quad (8)$$

where $h(x)$ is an asymptotic hadronic field operator.¹

Since $k^\alpha I_\alpha^a(x) = 0$, and the final hadrons are colorless particles, we get:

$$n_{\alpha\beta}^{a'b'}(k) = (-g_{\alpha\beta} k^2 + k_\alpha k_\beta) \delta^{a'b'} n_g(k^2) , \quad (9)$$

where dimensionless quantity $n_g(k^2)$ describes the average multiplicity of hadrons in the gluon jet with the virtuality k^2 . It is, of course, gauge invariant, and depends only on the virtuality k^2 .

It is useful to introduce the average multiplicity from the gluon jet whose *virtuality* p^2 *varies up to* k^2 :

$$N_g(k^2, Q_0^2) = \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} n_g(p^2) . \quad (10)$$

Very often $N_g(k^2)$ is erroneously called the average multiplicity of the gluon jet with *fixed virtuality* k^2 . This meaning should be addressed to $n_g(k^2)$ only.

The infrared cut-off Q_0 which separates perturbative and non-perturbative regions was introduced in (10). We will use the “conventional standard” value $Q_0 = 1$ GeV for our numerical estimates (see Section 4).

¹For half-spin hadrons, $D_h^-(x)$ is replaced by $S_h^-(x) = (i\hat{\partial} + m_h)D_h^-(x)$.

The quantity $n_g(k^2)$ cannot be calculated perturbatively. It is usually assumed that the average hadron multiplicity is proportional to $n_g(k^2, Q_0^2)$, i.e. the average multiplicity of (off-shell) partons with the “mass” Q_0 (the so-called local parton-hadron duality):

$$n_g(k^2) = n_g(k^2, Q_0^2) K(Q_0^2) , \quad (11)$$

where $K(Q_0^2)$ is a phenomenological energy-independent factor. The QCD evolution equations for both $n_g(k^2, Q_0^2)$ and $N_g(k^2, Q_0^2)$ are derived in Appendix A. Let us stress, however, that the main results of the present paper (see Sections 3, 4) *do not depend on explicit form* of the function $n_g(k^2)$.

In Eq. (5) the first factor of the integrand is given by

$$\Pi_{\mu\nu}^{ab}(q, k) = (-g^{\rho\sigma}) \Pi_{\rho\sigma;\mu\nu}^{ab}(q, k) , \quad (12)$$

where $\Pi_{\rho\sigma;\mu\nu}^{ab}(q, k)$ is the two-gluon irreducible part of the relevant discontinuity of the four-current correlation function²

$$\begin{aligned} \Pi_{\rho\sigma;\mu\nu}^{ab}(q, k) &= \iiint d^4x d^4y d^4z e^{iqx - ik(y-z)} \\ &\times \langle 0 | \left(T J_\rho^{\text{em}}(x) I_\mu^a(y) \right) \left(T J_\sigma^{\text{em}}(0) I_\nu^b(z) \right) | 0 \rangle . \end{aligned} \quad (13)$$

Here $J_\rho^{\text{em}}(x)$ is the electromagnetic current. The color current $I_\mu^a(x)$ was defined above (7).

In the first order in the strong coupling constant, $\Pi_{\mu\nu}^{ab}(q, k)$ is given by two diagrams presented in Fig. 2 and Fig. 3 normalized to the total e^+e^- rate. This quantity is gauge invariant:

$$k^\mu \Pi_{\mu\nu}^{ab}(q, k) = 0 . \quad (14)$$

Let us define

$$\begin{aligned} C_F \frac{\alpha_s(k^2)}{\pi k^2} E(Q^2, k^2) &= \frac{1}{(2\pi)^3 Q^2} \left(-k^2 \frac{\partial}{\partial k^2} \right) \int d(qk) \sqrt{(qk)^2 - Q^2 k^2} \\ &\times (-g^{\mu\nu}) \delta_{ab} \Pi_{\mu\nu}^{ab}(Q^2, k^2, qk) , \end{aligned} \quad (15)$$

where $C_F = (N_c^2 - 1)/2N_c$, and $N_c = 3$ is the number of colors. Then the average multiplicity in e^+e^- annihilation (5) looks like

$$N_{q\bar{q}}^h(Q^2, Q_0^2) = 2n_q + C_F \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{\pi} E(Q^2, k^2) N_g(k^2) . \quad (16)$$

²Note that $\Pi_{\rho\sigma;\mu\nu}^{ab}$ is proportional to δ^{ab} .

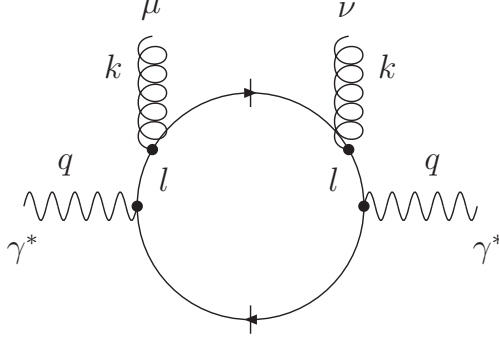


Figure 2: The inclusive distribution of the massive gluon jet (winding line) with the virtuality k^2 . The wavy line is the virtual photon with 4-momentum q . The solid lines are light quarks. The cut lines correspond to on-shell quarks.

The term $E(Q^2, k^2)$ is the inclusive spectrum of the gluon jet with the virtuality up to k^2 emitted by primary quarks.³

For the light quark case, the explicit form of $E(Q^2, k^2)$ was obtained in Ref. [13]. In terms of the variable

$$\sigma = \frac{k^2}{Q^2}, \quad (17)$$

we get:

$$\begin{aligned} E(\sigma) = & (1 + 2\sigma + 2\sigma^2) \ln \frac{1}{\sigma} - \frac{3 + 7\sigma}{2} (1 - \sigma) - \sigma(1 + \sigma) \left(\ln \frac{1}{\sigma} \right)^2 \\ & + 4\sigma(1 + \sigma) I(\sigma), \end{aligned} \quad (18)$$

where

$$I(\sigma) = \frac{\pi^2}{4} - \text{Li}_2(1 + \sigma), \quad (19)$$

and $\text{Li}_2(z)$ is the Euler dilogarithm.

³It was explained in detail in Ref. [13] that one should not consider this mechanism of hadron production via gluon jets as due to “a single cascading gluon”, as some authors believe [16]. That E is an inclusive spectrum of the gluon jets is seen, e.g., from the fact that the average number of gluon jets $\int (dk^2/k^2) E(Q^2, k^2) > 1$.

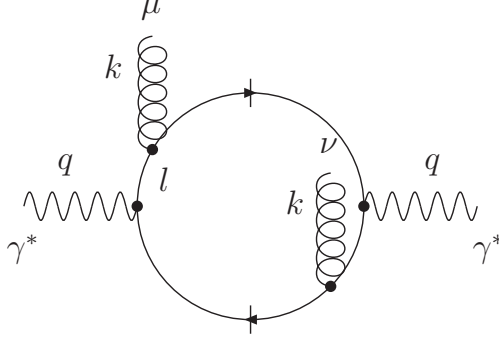


Figure 3: The interference diagram which also contributes to the inclusive distribution of gluon jets.

Let us introduce new variables

$$\eta = \ln \frac{Q^2}{k^2} \quad (20)$$

and

$$Y = \ln \frac{Q^2}{Q_0^2} , \quad (21)$$

as well as the notation

$$\hat{N}_g(k^2) = C_F \frac{\alpha_s(k^2)}{\pi} N_g(k^2) . \quad (22)$$

Then Eq. (16) can be rewritten as

$$N_{q\bar{q}}(Y) = 2 n_q + \int_0^Y d\eta \hat{N}_g(Y - \eta) E(\eta) = 2 n_q + N_q(Y) . \quad (23)$$

The physical meaning of the function

$$N_q(Y) = \int_0^Y d\eta \hat{N}_g(Y - \eta) E(\eta) \quad (24)$$

in Eq. (23) is the following. It describes the average number of hadrons produced from virtual gluon jets emitted by primary quark (antiquark) of

the type q . In other words, it is the multiplicity in $q\bar{q}$ event except for the multiplicity of the decay products of these quarks at the final stage of hadronization (the first term in (23)).

The function $E(\eta)$ is presented in Fig. 4. It has the asymptotics

$$E(\eta)|_{\eta \rightarrow \infty} = E^{\text{asym}}(\eta) = \eta - \frac{3}{2}, \quad (25)$$

The derivative of $E(\eta)$ is positive for all η , as one can see in Fig. 4. It follows from the relation $\partial N_{q\bar{q}}(Y)/\partial Y = \int_0^Y d\eta \hat{N}_g(\eta) \partial E(Y-\eta)/\partial Y$ that the associative multiplicity $N_{q\bar{q}}(Q)$ (23) is a monotone increasing function of Q for any positive function $N_g(k^2)$.

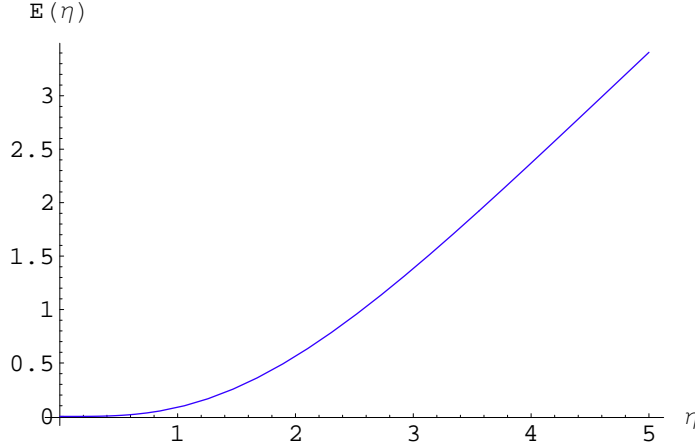


Figure 4: The function $E(\eta)$.

3 Multiplicity difference in QCD

Now let us consider the difference between multiplicities in events induced by the light and heavy quarks, δ_{Ql} , which is defined by Eq. (1). The following representation was found in Ref. [13]:

$$\delta_{Ql}^{\text{QCD}} = 2(n_Q - n_l) - \Delta N_Q(Y_m). \quad (26)$$

Here the new notation is introduced,

$$\Delta N_Q(Y_m) = N_q - N_Q = \int_{-\infty}^{Y_m} dy \hat{N}_g(Y_m - y) \Delta E_Q(y), \quad (27)$$

as well as variables

$$y = \ln \frac{m_Q^2}{k^2} , \quad (28)$$

and

$$Y_m = \ln \frac{m_Q^2}{Q_0^2} . \quad (29)$$

An important result which was obtained in Ref. [13] is that the function

$$\Delta E_Q = E - E_Q \quad (30)$$

depends only on the variable

$$\rho = \exp(-y) , \quad (31)$$

but not on the energy Q . The explicit form of ΔE_Q is the following:

$$\begin{aligned} \Delta E_Q(\rho) &= (1 - 3\rho + \frac{7}{2}\rho^2) \ln \frac{1}{\rho} + \rho(7\rho - 20) J(\rho) + \frac{20}{\rho - 4} [1 - J(\rho)] \\ &+ 7\rho + \frac{9}{2} , \end{aligned} \quad (32)$$

where

$$J(\rho) = \begin{cases} \sqrt{\frac{\rho}{\rho-4}} \ln \left(\frac{\sqrt{\rho} + \sqrt{\rho-4}}{2} \right) , & \rho > 4 , \\ 1 , & \rho = 4 , \\ \sqrt{\frac{\rho}{4-\rho}} \arctan \left(\frac{\sqrt{4-\rho}}{\rho} \right) , & \rho < 4 . \end{cases} \quad (33)$$

Since $\Delta E_Q(y)$ has the asymptotics

$$\Delta E_Q(y) \Big|_{y \rightarrow -\infty} \simeq \frac{11}{3} \exp(-|y|) , \quad (34)$$

the integral in Eq. (27) converges rapidly at $y \rightarrow -\infty$. The function $\Delta E_Q(y)$ is shown in Fig. 5. We find that

$$\Delta E_Q(y) \Big|_{y \rightarrow \infty} = \Delta E_Q^{\text{asym}}(y) = y - \frac{3}{2} . \quad (35)$$

Another important relation comes from (25) and (35):

$$\Delta E_Q(y-1) - E(y) \Big|_{y \rightarrow \infty} \simeq \frac{5}{2} \sqrt{e} \ln 2 \exp(-y/2) . \quad (36)$$

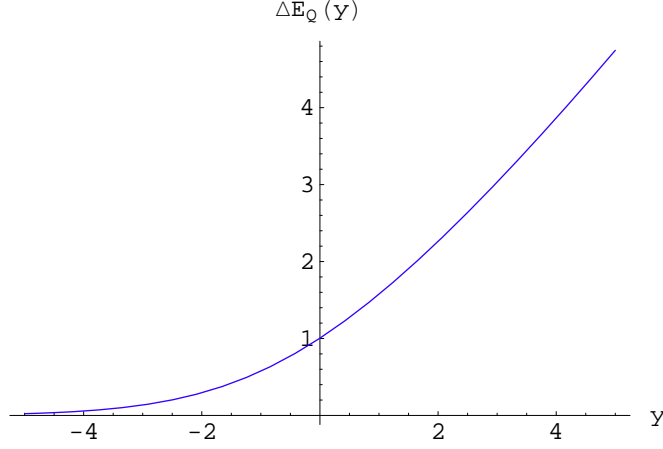


Figure 5: The function $\Delta E_Q(y)$.

In other words,

$$\Delta E_Q(y) \simeq E(y+1) \quad (37)$$

at large y .

If one puts $\Delta E_Q(y) = E(y+1)$, then (neglecting the contribution from the region $y < -1$):

$$\Delta N_Q = N_{l\bar{l}}(m_Q^2 e) - 2n_l. \quad (38)$$

Correspondingly, the approximate expression for δ_{Ql} is of the form:

$$\delta_{Ql}^{\text{appr}} = 2n_Q - N_{l\bar{l}}(m_Q^2 e) = \delta_{Ql}^{\text{MLLA}}, \quad (39)$$

where $\delta_{Ql}^{\text{MLLA}}$ is the MLLA prediction for the multiplicity difference [6]. We would remind that the function $N_{l\bar{l}}(Q)$ describes the hadron multiplicity in light quark events at the collision energy Q .

However, expression (37) is very far from the exact one in the region $y < Y_m$,⁴ as it is clearly seen in Fig. 6. That is why, there is a large difference between $\delta_{Ql}^{\text{MLLA}}$ (39) and QCD expression δ_{Ql}^{QCD} (26).

⁴For the beauty case, one has $Y_m \lesssim 3.2$.

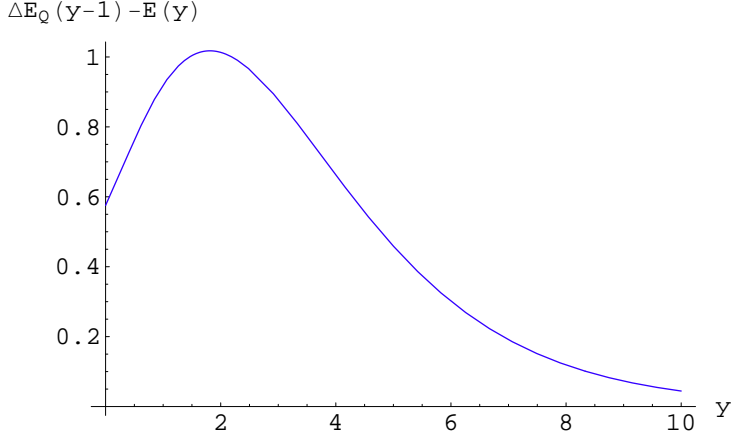


Figure 6: The difference $\Delta E_Q(y-1) - E(y)$ as a function of the variable y .

4 Multiplicity difference: upper and lower bounds

In this section we will show that the difference between $\delta_{Ql}^{\text{MLLA}}$ (39) and δ_{Ql}^{QCD} is indeed numerically large, and also obtain both upper and lower bounds on δ_{bl} .

It is convenient to represent expression for ΔN_Q (27) in the form:

$$\begin{aligned}
\Delta N_Q(Y_m) &= \int_0^{Y_m+1} dy \hat{N}_g(Y_m+1-y) E(y) \\
&+ \int_{-\infty}^{-1} dy \hat{N}_g(Y_m-y) \Delta E_Q(y) \\
&+ \int_0^{Y_m+1} dy \hat{N}_g(Y_m+1-y) [\Delta E_Q(y-1) - E(y)] \\
&\equiv [N_{l\bar{l}}(m_Q^2 e) - 2 n_l] + \delta N_Q^{(1)}(Y_m) + \delta N_Q^{(2)}(Y_m) , \quad (40)
\end{aligned}$$

that results in the formula (see Eq. (26))

$$\begin{aligned}\delta_{Ql}^{\text{QCD}} &= 2n_Q - N_{l\bar{l}}(m_Q^2 e) - \delta N_Q^{(1)}(Y_m) - \delta N_Q^{(2)}(Y_m) \\ &= \delta_{Ql}^{(appr)} - \delta N_Q^{(1)}(Y_m) - \delta N_Q^{(2)}(Y_m) .\end{aligned}\quad (41)$$

Here we have introduced notations

$$N_Q^{(1)}(Y_m) = \int_{-\infty}^{-1} dy \hat{N}_g(Y_m - y) \Delta E_Q(y) , \quad (42)$$

and

$$N_Q^{(2)}(Y_m) = \int_0^{Y_m+1} dy \hat{N}_g(Y_m + 1 - y) [\Delta E_Q(y - 1) - E(y)] . \quad (43)$$

Note that both $N_Q^{(2)}(Y_m)$ and $N_Q^{(1)}(Y_m)$ are positive since $\Delta E_Q(y) > 0$ at all y and $\Delta E_Q(y - 1) - E(y) > 0$ at $y \geq 0$ (see Fig. 5 and Fig. 6).

In order to exploit the corrected data on $N_{l\bar{l}}(Q^2)$ at $Q = 8$ GeV,

$$N_{l\bar{l}}(8.0 \text{ GeV}) = 6.70 \pm 0.34 , \quad (44)$$

we assume that $m_b = 4.85$ GeV, that corresponds to $m_b \sqrt{e} = 8$ GeV.

The estimates show that the dominant correction to δ_{Ql}^{QCD} is $\delta N_Q^{(2)}$, not $\delta N_Q^{(1)}$. To calculate a lower bound for $\delta N_b^{(2)}$, we will use the following inequality in the region $y \geq 0$:

$$\Delta E_Q(y) \geq E(y + \Delta y_Q) . \quad (45)$$

The quantity Δy_Q is a solution of the equation

$$\Delta E_Q(Y_m) = E(Y_m + \Delta y_Q) , \quad (46)$$

where Y_m is defined above (29). Then we get from Eqs. (43) and (45):

$$\delta N_Q^{(2)} \geq N_{l\bar{l}}(Y_m + \Delta y_Q) - N_{l\bar{l}}(Y_m + 1) - \int_0^{\Delta y_Q - 1} dy \hat{N}_g(Y_m + \Delta y_Q - y) E(y) . \quad (47)$$

For our further estimates, we need to know the hadron multiplicity in the light quark events in the energy interval $2.5 \text{ GeV} \leq Q \leq 28 \text{ GeV}$. By fitting

the data on hadron multiplicity in the light quark events at low Q , we get the expression:

$$N_{l\bar{l}}(Q^2) = 2.07 + 1.11 \ln Q + 0.54 \ln^2 Q . \quad (48)$$

Putting $Q_0 = 1$ GeV, we find $\Delta y_b = 1.61$. Taking into account that the last term in Eq. (47) is negligible,⁵ we get from (47), (48):

$$\delta N_b^{(2)} \geq 1.07 . \quad (49)$$

Correspondingly, our prediction accounting the revised data on the multiplicity in events induced by the light quarks,

$$\delta_{bl}^{\text{QCD}} \leq 2n_b - N_{l\bar{l}}(Y_m + \Delta y_b) = 3.33 \pm 0.38 , \quad (50)$$

appears to be lower than our previous result $\delta_{bl} = 3.68$ [13]. We used the experimental value

$$2n_b = 11.10 \pm 0.18 . \quad (51)$$

The error in measurements of $N_{l\bar{l}}$ was taken to be ± 0.34 . Let us stress that our upper bound (50) is very close to the present experimental value of δ_{bl}^{exp} (3).

Now let us derive a lower bound on δ_{bl}^{QCD} . To do this, let us start from Eq. (27). It is convenient to represent the integral in (27) as a sum of two terms:⁶

$$\begin{aligned} \Delta N_b &= \int_{-4}^{-1} dy \hat{N}_g(Y_b - y) \Delta E_b(y) + \int_{-1}^{Y_b} dy \hat{N}_g(Y_m - y) \Delta E_b(y) \\ &= \Delta N_b^{(1)} + \Delta N_b^{(2)} , \end{aligned} \quad (52)$$

where $Y_b = \ln(m_b^2/Q_0^2) \simeq 3.16$. Consider the first term in (52). One can check that

$$\Delta E(y) < 0.18 E(y + 5.8) \quad (53)$$

in the region $-4 < y < -1$, that leads to the inequality

$$\Delta N_b^{(1)} < 0.18 \int_{1.8}^{4.8} dy \hat{N}_g(Y_b + 5.8 - y) \Delta E_b(y) . \quad (54)$$

⁵Since $E(y) < 0.02$ in the region $0 \leq y \leq \Delta y_b - 1 = 0.61$.

⁶We take into account that the region $-\infty < y < -4$ gives a negligible contribution to ΔN_b .

The estimations show that $\hat{N}_g(Y_b + 5.8 - y) < 2 \hat{N}_g(4.8 - y)$ when y varies from 1.8 to 4.8. Thus, we get:

$$\Delta N_b^{(1)} < 0.36 [N_{\bar{u}}(Q = 11 \text{ GeV}) - N_{\bar{u}}(Q = 2.5 \text{ GeV})] = 1.54 \pm 0.17 . \quad (55)$$

The second term in (52) can be estimated by using the inequality

$$\Delta E(y) < 0.62 E(y + 3.5) \quad (56)$$

which is valid in the region $-1 < y < Y_b$. Then

$$\Delta N_b^{(2)} < 0.62 [N_{\bar{u}}(Q = 28 \text{ GeV}) - N_{\bar{u}}(Q = 3.5 \text{ GeV})] = 4.61 \pm 0.30 . \quad (57)$$

As a result, we obtain from Eqs. (26), (27) and (55), (57) the lower bound on δ_{bl}^{QCD} :

$$\delta_{bl}^{\text{QCD}} > 2.55 \pm 0.39 . \quad (58)$$

Fig. 7 demonstrates that our QCD predictions are very close to the average measurement $\delta_{bl}^{\text{exp}} = 3.12$.

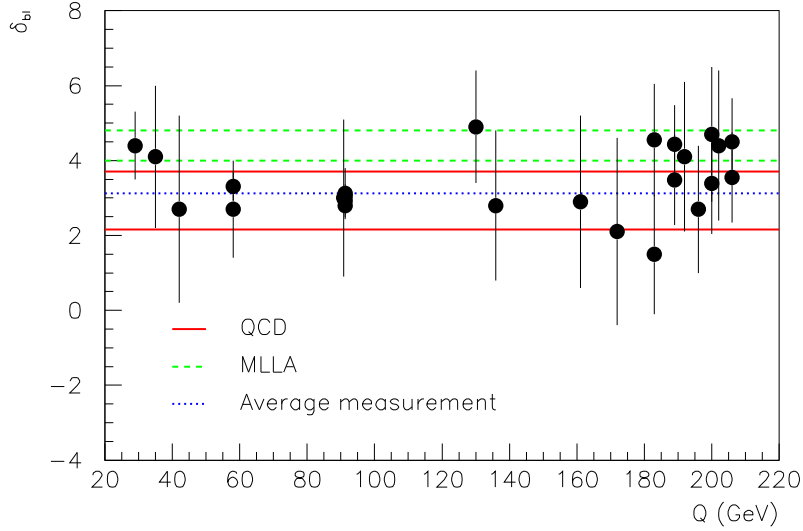


Figure 7: Our QCD result for δ_{bl} (a corridor between the two solid lines), and the MLLA prediction [16] (a corridor between the two dashed lines) vs. experimental data.

Our results can be compared with the recently published new MLLA results [16]:

$$\delta_{bl}^{\text{MLLA}} = 4.4 \pm 0.4 . \quad (59)$$

The next-to-MLLA results are:

$$\delta_{bl}^{\text{NMLLA}} = 2.6 \pm 0.4 , \quad (60)$$

$$\delta_{cl}^{\text{NMLLA}} = -0.1 \pm 0.4 . \quad (61)$$

Thus, the lowest-order MLLA expression (2) is not stable against higher order corrections. As it was said above, the formula which was used in the MLLA approach (39) is only *a part* of the exact QCD formula (1), (27) in the approximation $E(y) = \Delta E_Q(y - 1)$. This approximation is quite a rough one (see Fig. 6), and the deviation of the function $\Delta E_Q(y)$ from $\Delta E_Q^{\text{asym}}(y) = y - 3/2$, as well as the deviation of $E(y)$ from $E^{\text{asym}}(y) = y - 1/2$, cannot be neglected.

Note that the limit $y \rightarrow \infty$ means that the invariant mass of the gluon jet k^2 tends to zero, since $k^2 = Q^2 \exp(-y)$ (see (28)). We also think that using the argument k_\perp^2 in N_g instead of k^2 in the MLLA scheme (see [16] and references therein) has no reasons. Indeed, k is a time-like vector ($k^2 = k_0^2 - k_\perp^2 - k_\parallel^2 > 0$), and the gluon jet with large transverse momentum k_\perp has the small invariant mass k^2 . In such a jet, the multiplicity is small, since the phase space for secondary particles is actually defined by the invariant mass.

5 Conclusions

The formula for the difference between hadron multiplicities in e^+e^- annihilation into hadrons induced by light and heavy primary quarks (with Q is a type of a heavy quark) is derived:

$$\begin{aligned} \delta_{Ql}^{\text{QCD}} &= 2n_Q - N_{l\bar{l}}(m_Q^2 e) \\ &- \int_{Q_0^2}^{m_{Qe}^2} \frac{dk^2}{k^2} \hat{N}_g(k^2) \left[\Delta E_Q \left(\frac{m_Q^2}{k^2} \right) - E \left(\frac{m_{Qe}^2}{k^2} \right) \right] \\ &- \int_{m_{Qe}^2}^{\infty} \frac{dk^2}{k^2} \hat{N}_g(k^2) \Delta E_Q \left(\frac{m_Q^2}{k^2} \right) . \end{aligned} \quad (62)$$

Here $\hat{N}_g(k^2)$ describes the average number of charged hadrons in the gluon jet with the virtuality which varies up to k^2 , and $E, \Delta E_Q$ are known functions.

By using the data on the hadron multiplicity in light quark events $N_{l\bar{l}}$, corrected for the detector effects and initial state radiation effects [16], we have obtained from (62) the bounds:

$$2.2 < \delta_{bl}^{\text{QCD}} < 3.7 . \quad (63)$$

We would like to emphasize that our estimate does not depend on a specific choice of the function $N_g(k^2)$, and it is in a good agreement with the average experimental value $\delta_{bl}^{\text{exp}} = 3.12 \pm 0.14$.

Two last terms in (62), which are subtracted from the first one, are positive and numerically large. In particular, for the case of the beauty ($m_Q = m_b$, $n_Q = n_b$) the second term in (62) (dominating the third one) is equal to 1.1.

As a result, the deviation of the MLLA prediction,

$$\delta_{bl}^{\text{MLLA}} = 2n_b - N_{l\bar{l}}(m_b^2 e) , \quad (64)$$

from the exact expression,⁷

$$\delta_{bl}^{\text{QCD}} = 2(n_b - n_l) - \int_{Q_0^2}^{m_b^2} \frac{dk^2}{k^2} \hat{N}_g(k^2) \Delta E_Q \left(\frac{m_b^2}{k^2} \right) , \quad (65)$$

appears to be quite significant.

Let us also mention that our numerical predictions for the charm quark case,

$$\begin{aligned} \delta_{cl}^{\text{QCD}}(Q = 91 \text{ GeV}) &= 1.01 , \\ \delta_{cl}^{\text{QCD}}(Q = 170 \text{ GeV}) &= 0.99 , \end{aligned} \quad (66)$$

were derived in [13] *before the precise measurements of δ_{cl} were made* [11].⁸ As one can see, our value (66) is in a nice agreement with the average experimental value (4) (see also Fig. 7).

Some part of the results of this paper was published in [18]. We are indebted to W. Ochs, the correspondence with whom stimulated, to some extent, the appearance of the present paper.

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⁷This formula is an equivalent compact form of Eq. (62) for $Q = b$.

⁸In the low energy measurements [4, 8], the total error of δ_{cl} was about ± 1.5 .

Appendix A

Let us consider the average multiplicity of off-shell quanta with the “mass” Q_0 in a gluon jet whose invariant mass is p^2 . It obeys the following integral equation [19, 20]:

$$\begin{aligned} n_g(p^2, Q_0^2) &= p^2 \delta(p^2 - Q_0^2) + \int_{Q_0^2}^{p^2} \frac{dl^2}{l^2} \int_0^1 dz \theta(zp^2 - l^2) \theta(zk^2 - p^2) \\ &\times \frac{\alpha_s(p^2 z)}{2\pi} \hat{P}_{gg}(z) n_g(l^2, Q_0^2) . \end{aligned} \quad (\text{A.1})$$

Here k^2 is the virtuality of the parent quark which emits this gluon jet, and $\hat{P}_{gg}(z)$ is Altarelli-Parisi time-like decay function.

The inequality

$$z \geq \frac{l^2}{p^2} \quad (\text{A.2})$$

in Eq. (A.1) is a kinematical bound, while the bound

$$z \geq \frac{p^2}{k^2} \quad (\text{A.3})$$

is a dynamical one. The latter is nothing but the angle ordering rewritten in terms of momentum fractions and virtualities (see Refs. [19], [20]).⁹

Assuming that parton shower develops mainly via *soft* gluons, one can put in (A.1)

$$\hat{P}_{gg}(z) \Big|_{z \ll 1} \simeq 2C_A \frac{1}{z} . \quad (\text{A.4})$$

Then the sequence of equations [21],

$$\begin{aligned} N_g(k^2, Q_0^2) - 1 &= C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \int_{Q_0^2}^{p^2} \frac{dl^2}{l^2} \int_0^1 \frac{dz}{z} \theta(zp^2 - l^2) \theta(zk^2 - p^2) \frac{\alpha_s(zp^2)}{\pi} n_g(l^2, Q_0^2) \\ &= C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(\sqrt{k^2 Q_0^2} - p^2) \int_{Q_0^2}^{p^2} \frac{dl^2}{l^2} \int_0^1 \frac{dz}{z} \theta(zp^2 - l^2) \frac{\alpha_s(zp^2)}{\pi} n_g(l^2, Q_0^2) \end{aligned}$$

⁹Namely, the emission angle of the secondary gluon with the virtuality l^2 is less than the angle of the primary gluon emission off the parent quark.

$$\begin{aligned}
& + C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(p^2 - \sqrt{k^2 Q_0^2}) \left[\int_{Q_0^2}^{p^2} \frac{dl^2}{l^2} \theta(p^4 - k^2 l^2) \int_0^1 \frac{dz}{z} \theta(z k^2 - p^2) \right. \\
& + \left. \int_{Q_0^2}^{p^2} \frac{dl^2}{l^2} \theta(k^2 l^2 - p^4) \int_0^1 \frac{dz}{z} \theta(z p^2 - l^2) \right] \frac{\alpha_s(z p^2)}{\pi} n_g(l^2, Q_0^2) \\
& = C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(\sqrt{k^2 Q_0^2} - p^2) \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\
& + C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(p^2 - \sqrt{k^2 Q_0^2}) \left[\int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \theta(r^2 k^2 - p^4) \int_{Q_0^2}^{r^2} \frac{dl^2}{l^2} \theta(p^4 - k^2 l^2) \right. \\
& + \left. \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \theta(r^2 k^2 - p^4) \int_{Q_0^2}^{r^2} \frac{dl^2}{l^2} \theta(k^2 l^2 - p^4) \right] \frac{\alpha_s(r^2)}{\pi} n_g(l^2, Q_0^2) \\
& = C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(\sqrt{k^2 Q_0^2} - p^2) \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\
& + C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(p^2 - \sqrt{k^2 Q_0^2}) \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \theta(k^2 r^2 - p^4) \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\
& = C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\
& - C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \theta(p^2 - \sqrt{k^2 Q_0^2}) \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \theta(p^4 - k^2 r^2) \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\
& = C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{2\pi} N_g(r^2, Q_0^2) , \tag{A.5}
\end{aligned}$$

results in the following formula for $N_g(k^2, Q_0^2)$:

$$N_g(k^2, Q_0^2) = 1 + C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{2\pi} N_g(r^2, Q_0^2) . \tag{A.6}$$

From (A.6) we obtain the differential equation:

$$\left(k^2 \frac{d}{dk^2} \right)^2 N_g(k^2, Q_0^2) = C_A \frac{\alpha_s(k^2)}{2\pi} N_g(k^2, Q_0^2) , \tag{A.7}$$

with the boundary conditions

$$N_g(k^2, Q_0^2) \Big|_{k^2=Q_0^2} = 1, \quad k^2 \frac{d}{dk^2} N_g(k^2, Q_0^2) \Big|_{k^2=Q_0^2} = 0 . \tag{A.8}$$

This equation has the solution:

$$\begin{aligned}
N_g(k^2, Q_0^2) &= \sqrt{\frac{2C_A}{\pi b} \ln \frac{k^2}{\Lambda^2}} \left[K_1 \left(\sqrt{\frac{2C_A}{\pi b} \ln \frac{k^2}{\Lambda^2}} \right) I_0 \left(\sqrt{\frac{2C_A}{\pi b} \ln \frac{Q_0^2}{\Lambda^2}} \right) \right. \\
&\quad \left. + I_1 \left(\sqrt{\frac{2C_A}{\pi b} \ln \frac{k^2}{\Lambda^2}} \right) K_0 \left(\sqrt{\frac{2C_A}{\pi b} \ln \frac{Q_0^2}{\Lambda^2}} \right) \right], \quad (\text{A.9})
\end{aligned}$$

with the asymptotics

$$N_g(k^2, Q_0^2)|_{k^2 \gg Q_0^2} \simeq \exp \left(\sqrt{\frac{2C_A}{\pi b} \ln \frac{k^2}{\Lambda^2}} \right). \quad (\text{A.10})$$

Here $b = (33 - 2N_f)/12\pi$, where N_f is a number of flavors.

Let us stress that the equation for *an isolated* gluon jet (which has no parent parton with virtuality k^2) would be of the form:

$$\begin{aligned}
n_g^{\text{isol}}(p^2, Q_0^2) &= p^2 \delta(p^2 - Q_0^2) + \int_{Q_0^2}^{p^2} \frac{dl^2}{l^2} \int_0^1 dz \theta(zp^2 - l^2) \\
&\quad \times \frac{\alpha_s(p^2 z)}{2\pi} \hat{P}_{gg}(z) n_g(l^2, Q_0^2), \quad (\text{A.11})
\end{aligned}$$

that leads to the formula:

$$N_g(k^2, Q_0^2) = 1 + 2C_A \int_{Q_0^2}^{k^2} \frac{dp^2}{p^2} \int_{Q_0^2}^{p^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{2\pi} N_g(r^2, Q_0^2). \quad (\text{A.12})$$

This equation results in a wrong expression which does not take into account the interference effects:

$$N_g(k^2, Q_0^2)|_{k^2 \gg Q_0^2} \simeq \exp \left(2\sqrt{\frac{C_A}{\pi b} \ln \frac{k^2}{\Lambda^2}} \right). \quad (\text{A.13})$$

Appendix B

Now we will reproduce the asymptotic relation between the average multiplicity in the light quark event and that in the gluon jet by using the Altarelli-Parisi decay functions. Let l be a 4-momentum of the primary quark which

emits a massive gluon jet. The ladder diagram in Fig. 2 leads to the equation

$$N_{\bar{l}l}(Q^2, Q_0^2) \Big|_{Q^2 \gg Q_0^2} = \int_{Q_0^2}^{Q^2} \frac{dl^2}{l^2} \int_{Q_0^2/l^2}^1 dz \frac{\alpha_s(zl^2)}{2\pi} \int_{Q_0^2}^{zl^2} \frac{dk^2}{k^2} \\ \times \hat{P}_{qg} \left(z, \frac{k^2}{l^2} \right) n_g(k^2, Q_0^2) \quad (\text{B.1})$$

(for simplicity, here and below we omit the contribution from the leading hadrons, $2n_q$).

In the leading logarithm approximation,

$$\hat{P}_{qg} \left(z, \frac{k^2}{l^2} \right) \simeq 2C_F \frac{1}{z}, \quad (\text{B.2})$$

one comes to the expression ($r^2 = zl^2$):

$$N_{\bar{l}l}(Q^2, Q_0^2) = C_F \int_{Q_0^2}^{Q^2} \frac{dl^2}{l^2} \int_{Q_0^2}^{l^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\ = C_F \int_{Q_0^2}^{Q^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \int_{r^2}^{Q^2} \frac{dl^2}{l^2} \\ = C_F \int_{Q_0^2}^{Q^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} \ln \frac{Q^2}{r^2} N_g(r^2, Q_0^2), \quad (\text{B.3})$$

where we have used the relation:

$$N_g(r^2, Q_0^2) = \int_{Q_0^2}^{r^2} \frac{dk^2}{k^2} n_g(k^2, Q_0^2). \quad (\text{B.4})$$

The integral equation for $N_g(r^2, Q_0^2)$ has been obtained in Appendix A (see Eq. (A.6)).

The formula (B.3) can be represented as¹⁰

$$N_{\bar{l}l}(Q^2, Q_0^2) = C_F \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{\pi} E^{\text{asym}}(Q^2, k^2) N_g(k^2, Q_0^2), \quad (\text{B.5})$$

with the function $E^{\text{asym}}(Q^2, k^2)$ defined above (25). It is worth to compare this approximate expression with our exact formula (16).

¹⁰We have added the non-leading term $-1/2$ to $\ln(Q^2/r^2)$ in deriving (B.5) from (B.3).

At large Q^2 , we obtain from (B.3) and (A.10) the well-known *asymptotic* relation:

$$N_{l\bar{l}}(Q^2, Q_0^2) \Big|_{Q^2 \gg Q_0^2} \simeq \frac{2C_F}{C_A} N_g(Q^2, Q_0^2) . \quad (\text{B.6})$$

Remember that $N_g(Q^2, Q_0^2)$ describes the average number of virtual partons in the gluon jet whose invariant mass *varies from* Q_0 *up to* Q .

Our formula (16) should be reproduced from the starting equation (B.1) provided:

1. Contributions from both the ladder (Fig. 2) *and interference* diagrams (Fig. 3) are added together in deriving the expression for $N_{l\bar{l}}(Q^2, Q_0^2)$
2. Both non-singular terms in variable z and power corrections $O(k^2/l^2)$ are taken into account in $\hat{P}_{qg}(z, k^2/l^2)$

Appendix C

The average number of hadrons in e^+e^- annihilation is, of course, gauge invariant quantity. However, in perturbative QCD we calculate the multiplicity of *virtual partons*. *A priori* one can not be sure that it does not depend on a gauge.

Unfortunately, this important problem was not studied to a considerable extent by other authors. As a few exceptions, let us mention Refs. [22] and [23]. In the first paper the gauge invariance of the multiplicity in e^+e^- annihilation has been checked in one-loop approximation. In the second one the gauge dependence was considered for a fixed coupling constant and without accounting for interference effects.

Below we will analyze a possible gauge dependence of the partonic multiplicity in the case of light primary quark in a class of axial gauges. It is given by Eq. (B.1) in the gauge $n_\mu A^\mu = 0$ with the gauge vector

$$n_\mu = \frac{1}{\sqrt{2}}(1, 0, -1) \quad (\text{C.1})$$

(z -axis is chosen along a 3-momentum of a primary quark in the c.m.s. of colliding leptons). The argument of the decay function $\hat{P}_{qg}(z)$ in (B.1) is the ratio

$$z = \frac{kn}{ln} . \quad (\text{C.2})$$

The emission of the gluon jets from the primary antiquark is suppressed in this gauge (C.1).

Analogously, in the gauge

$$n_\mu = \frac{1}{\sqrt{2}}(1, 0, 1) \quad (\text{C.3})$$

the massive gluon jet is emitted by the antiquark while its emission from the quark is suppressed.

Let us choose the gauge in which both quark and antiquark make comparable contributions to the emission:

$$n_\mu = (1, 0, 0) . \quad (\text{C.4})$$

Then Eq. (B.1) is modified as follows:

$$\begin{aligned} N_{l\bar{l}}(Q^2, Q_0^2) &= \int_{Q_0^2}^{Q^2} \frac{dl^2}{l^2} \int_{Q_0^2/l^2}^1 dz \frac{\alpha_s(zl^2)}{2\pi} \int_{Q_0^2}^{zl^2} \frac{dk^2}{k^2} \hat{P}_{qg}(z) n_g(k^2, Q_0^2) \\ &+ \int_{Q_0^2}^{Q^2} \frac{dl^2}{l^2} \int_{Q_0^2/l^2}^1 dz \frac{\alpha_s(zl^2)}{2\pi} \int_{Q_0^2}^{zl^2} \frac{dk^2}{k^2} \hat{P}_{\bar{q}g}(z) n_g(k^2, Q_0^2) , \end{aligned} \quad (\text{C.5})$$

where

$$\hat{P}_{qg}(z) = \hat{P}_{\bar{q}g}(z) \simeq 2C_A \frac{1}{z + \frac{1}{Q^2}} \quad (\text{C.6})$$

at small z . These relations mean that the massive jets are emitted by the quark and antiquark with the same probability in the gauge (C.4).

By omitting non-leading terms, we get from (C.5), (C.6) the expression,

$$\begin{aligned} N_{l\bar{l}}(Q^2, Q_0^2) &= 2C_F \int_{Q_0^2}^{Q^2} \frac{dl^2}{l^2} \int_{Q_0^2}^{l^2} \frac{dr^2}{r^2 + \frac{l^4}{Q^2}} \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \\ &= 2C_F \int_{Q_0^2}^{Q^2} dr^2 \frac{\alpha_s(r^2)}{\pi} N_g(r^2, Q_0^2) \int_{r^2}^{Q^2} \frac{dl^2}{l^2} \frac{1}{r^2 + \frac{l^4}{Q^2}} \\ &= C_F \int_{Q_0^2}^{Q^2} \frac{dr^2}{r^2} \frac{\alpha_s(r^2)}{\pi} \ln \frac{Q^2}{r^2} N_g(r^2, Q_0^2) , \end{aligned} \quad (\text{C.7})$$

which coincides with formula (B.3).

Note that in the gauge (C.4) the decay function in evolution equation (A.1) is of the form:

$$\hat{P}_{gg}(z) \simeq 2C_A \frac{1}{z + \frac{p^2}{Q^2}} . \quad (\text{C.8})$$

This results in the effective cut on the integration variable z from below:

$$z \geq \frac{p^2}{Q^2} . \quad (\text{C.9})$$

As one can see from (A.3) and (C.9), it is the dynamical bound (A.3) that smoothes out the singularity of $\hat{P}_{gg}(z)$ at $z = 0$, but not the bound (C.9) arising from the gauge vector. The latter can be safely omitted in Eq. (A.1).

Thus, we conclude that both the relation of the light quark multiplicity $N_{l\bar{l}}(k^2, Q_0^2)$ with the gluon multiplicity $N_g(Q^2, Q_0^2)$, and the evolution equation for $N_g(k^2, Q_0^2)$ do not depend on the gauge vector n_μ . This conclusion is also valid for a general case, $n_\mu = (n_0, 0, n_{||})$, where $n_0 \neq \pm n_{||}$ [19, 20]. Note that the proof of the gauge invariance needs an account of the destructive interference in the emission of the gluon jets, that leads to the condition (A.3).

References

- [1] Kartvelishvili V.G., Likhoded A.R., Petrov V.A. // Phys. Lett. B. 1978. V. 78. P. 615.
- [2] A.I. Akhiezer and V.B Berestetskii, *Quantum electrodynamics*, 2nd ed., New York, NY: Interscience, 1965.
- [3] Proc. of the 2005 International Linear Collider Workshop (LCWS 05), Stanford, California, USA, March 18-22, 2005.
- [4] MARK II Collaboration: P.C. Rowson *et al.*, Rev. Lett. **54** (1985) 2580;
- [5] A.V. Kisselev, V.A. Petrov, and O.P. Yuschenko, Z. Phys. C **41** (1988) 521.
- [6] B.A. Schumm, Yu.L. Dokshitzer, V.A. Khoze, and D.S. Koetke, Phys. Rev. Lett. **69** (1992) 3025.

- [7] DELCO Collaboration: M. Sakuda *et al.*, Phys. Lett. B **161** (1985) 412.
- [8] TPC Collaboration: H. Aihara *et al.*, Phys. Lett. B **184** (1987) 299;
TASSO Collaboration: W. Brauschweig, *et al.*, Z. Phys. C **42** (1989) 17;
TASSO Collaboration: M. Althoff *et al.*, Phys. Lett. B **42** (1984) 243.
- [9] TOPAZ Collaboration: K. Nagai *et al.*, Phys. Lett. B **278** (1992) 506;
VENUS Collaboration: K. Okabe *et al.*, Phys. Lett. B **423** (1998) 407.
- [10] MARK II Collaboration: B.A. Schumm *et al.*, Phys. Rev. D **646** (1992) 453;
DELPHI Collaboration: P. Abreu *et al.*, Phys. Lett. B **347** (1995) 447.
- [11] OPAL Collaboration: R. Akers *et al.*, Phys. Lett. B **352** (1995) 176;
SLD Collaboration: K. Abe *et al.*, Phys. Lett. B **386** (1996) 475; SLD
Collaboration: K. Abe *et al.*, Phys. Rev. D **69** (2004) 072003.
- [12] DELPHI Collaboration: P. Abreu *et al.*, Phys. Lett. B **479** (2000) 118;
Erratum, *ibid* B **492** (2000) 398; OPAL Collaboration: G. Abbiendi *et al.*,
Phys. Lett. B **550** (2002) 33.
- [13] V.A. Petrov and A.V. Kisselev, Z. Phys. C **66** (1995) 453; Nucl. Phys.
(Proc. Suppl.) B **39**, C (1995) 364.
- [14] DELPHI Collaboration: J. Chrin *et al.*, in *Proc. of the 27-th International
Conference on High Energy Physics*, Glasgow, UK, 20-27 July
1994, eds. P.J. Bussey and I.G. Knowles, p. 893.
- [15] W. Metzger, in *Proc. of the International Europhysics Conference on
High Energy Physics*, Brussel, Belgium, 27 July- 2 August 1995, eds.
J. Lemonne, C. Vander Velde and F. Verbeure (World Sientific, Singa-
pore), p. 323.
- [16] Yu.L. Dokshitzer, F. Fabbri, V.A. Khoze, and W. Ochs, Eur. Phys. J.
C **45** (2006) 387.
- [17] N.N. Bogolyubov and D.V. Shirkov, *Introduction to the theory of quan-
tized fields*, 3rd ed., New York, NY: Wiley, 1980; N.P. Konopleva and
V.N. Popov, *Gauge fields*, Chur: Harwood, 1981 (Translated from Rus-
sian).

- [18] Kisselev A.V., Petrov V.A. // Eur. Phys. J. C. 2006. V. 50. P. 21.
- [19] A.V. Kisselev, JETP Letters **35** (1982) 215; A.V. Kisselev, Preprint IHEP 82-164, Serpukhov, 1982.
- [20] A.V. Kisselev and V.A. Petrov, Particles & Nuclei **19** (1988) 21.
- [21] A.V. Kisselev, *Doctor Thesis*, Protvino, 1995 (in Russian, unpublished).
- [22] W. Furmanski, R. Petronzio, and S. Pokorski, Nucl. Phys. B **135** (1979) 253.
- [23] A. Bassetto, M. Ciafaloni, and G. Marchesini, Nucl. Phys. B **163** (1982) 477.